Physics 216 Mathematical Physics
Quiz 1, October 25 2013, Time: 90 minutes

1. Let $\overrightarrow{v}, \overrightarrow{w}$ be three dimensional complex vectors given by

$$\overrightarrow{v} = (1, 3-i, 2+3i), \qquad \overrightarrow{w} = (-4-4i, 1+2i, 3-i)$$

Compute the inner products $(\overrightarrow{v}, \overrightarrow{v})$, $(\overrightarrow{w}, \overrightarrow{w})$, $(\overrightarrow{v}, \overrightarrow{w})$ and verify the Schwarz's inequality

$$|(\overrightarrow{v}, \overrightarrow{w})|^2 \leq (\overrightarrow{v}, \overrightarrow{v})(\overrightarrow{w}, \overrightarrow{w})$$

2. Under what conditions on the scalar x do the vectors $(0, 1, x)$, $(x, 0, 1)$, $(x, 1, 1+x)$ form a basis of a three dimensional vector space.

3. Which of the following three definitions of transformations on the vector space of polynomials P give linear transformations

   (a) $Tp(x) = p\left(x^2\right)$
   (b) $Tp(x) = (p(x))^2$
   (c) $Tp(x) = x^2 p(x)$

4. Consider the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$. Find the eigenvalues and eigenvectors of A. Determine the diagonalizing matrix P.

5. Let R be the the transformation on n dimensional vectors such that $\overrightarrow{x}' = R\overrightarrow{x}$. Find the condition on the matrices R such that the length of the vector $(\overrightarrow{x}, \overrightarrow{x})$ remains unchanged.

6. Consider the vector field $F = \left(x + y^2\right)\overrightarrow{i} + (xy - 1)\overrightarrow{j}$. Evaluate the line integral $\oint_C \overrightarrow{F}.\overrightarrow{dr}$ first for the circle with the center anywhere on the $x-$axis and second for the square of vertices $(0,0)$, $(1,-1)$, $(2,0)$, $(1,1)$. Is the vector $\overrightarrow{F}$ conservative.

7. Evaluate the surface integral $\int_S \overrightarrow{F}.\overrightarrow{da}$ for $\overrightarrow{F} = x^2\overrightarrow{i} + y^2\overrightarrow{j} + z^2\overrightarrow{k}$ and S consists the faces of the unit cube $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$.

8. Find $\overrightarrow{\nabla}.\overrightarrow{F}$ for $\overrightarrow{F} = \rho^3\widehat{\rho} + \rho^2 \sin\varphi\widehat{\varphi} + z^2\widehat{z}$ in cylindrical coordinates.

9. Show the identity $\overrightarrow{\nabla} \times \left(\overrightarrow{\nabla} \times \overrightarrow{V}\right) = \overrightarrow{\nabla}\left(\overrightarrow{\nabla} \cdot \overrightarrow{V}\right) - \nabla^2\overrightarrow{V}$.

10. Let

$$\delta_n(x) = \begin{cases} 0 & |x| > \frac{1}{2n} \\ n & |x| < \frac{1}{2n} \end{cases}$$

Evaluate the integral $\int_{-\infty}^{\infty} f(x)\,\delta_n(x-a)\,dx$.